An Econometric Investigation of the Influence of Transit Passes on Transit Users' Behaviour in Toronto

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Outline

- Research Question
- Data and Relevant Issues
- Modelling Technique Fitting Research Context and Data Limitation
- Joint Choice Model: Ownership and Frequency
- Joint Choice and Distance Travel Demand Model
- Empirical Model
- Conclusion

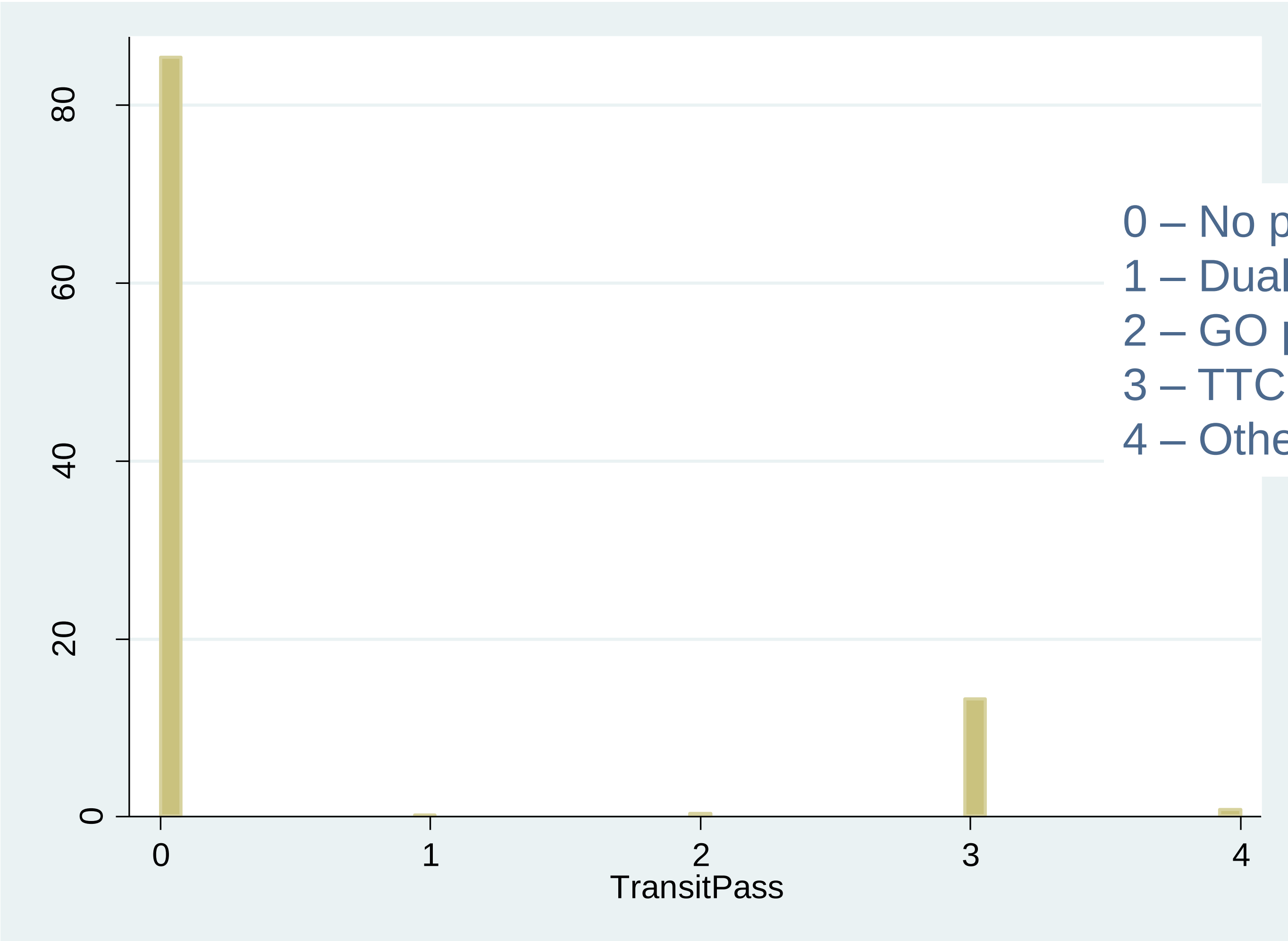
Research Question & Challenge

- Does a transit pass return more benefit than its usage (mobility tool): Toronto Transit Comission-TTC

- Comprehensive analysis requires data on transit usage behaviour of different fare class/type users:
  - Longitudinal survey of transit users
  - Stated Preference surveys

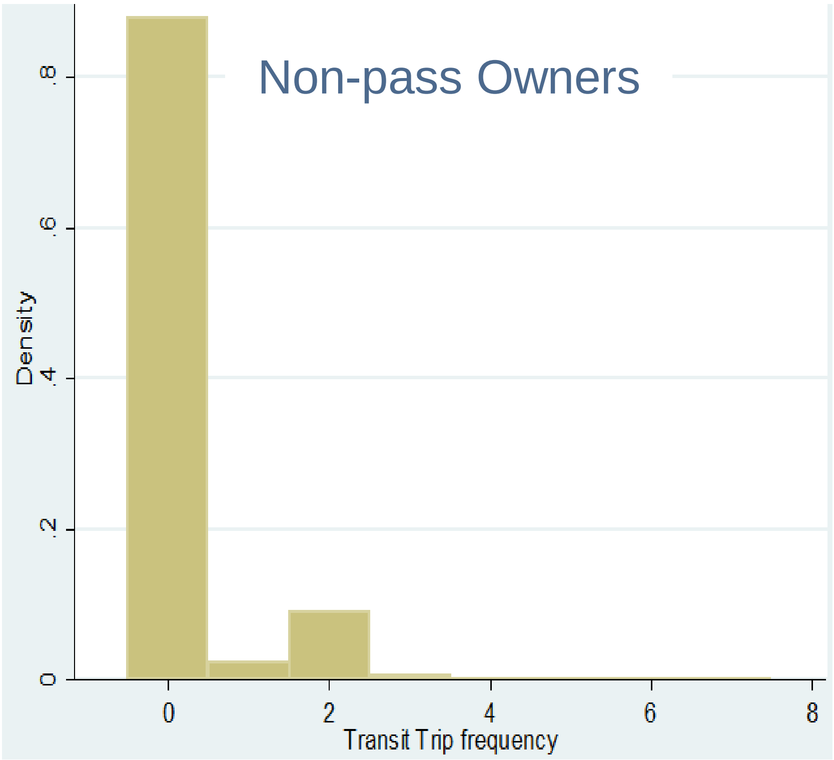
- However, we currently have one-day household travel diary survey: 5% Sample of Household Travel in the GTHA:
  - ✓The Transportation Tomorrow Survey (TTS)

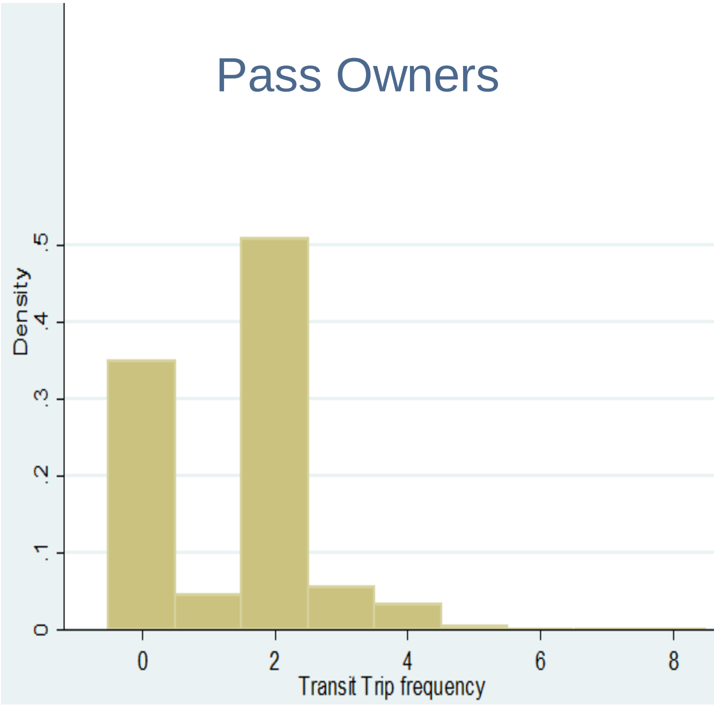
TTS Data: Transit pass ownership



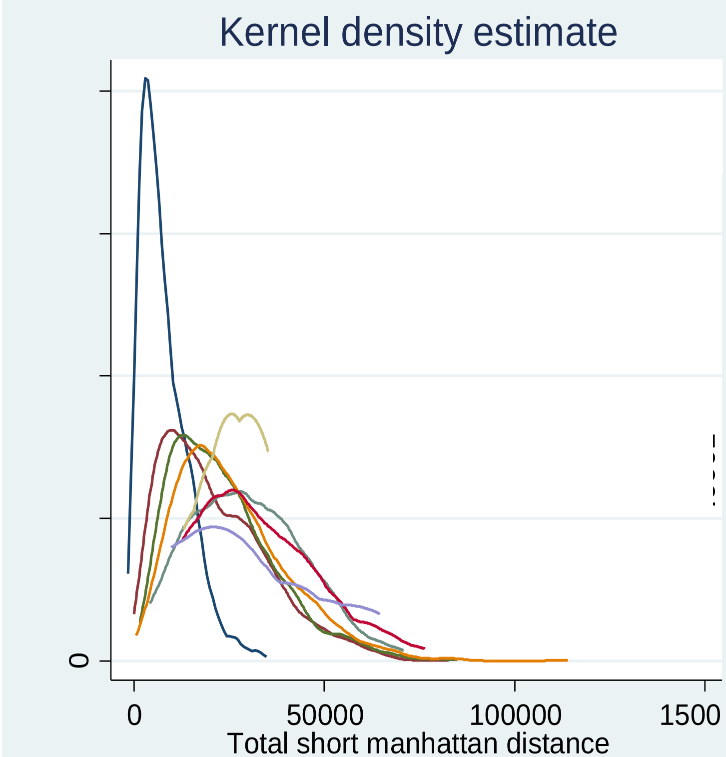
0 – No pass owning
1 – Dual transit passes
2 – GO pass
3 – TTC pass
4 – Other pass

TTS Data: Daily Transit Trip Frequency

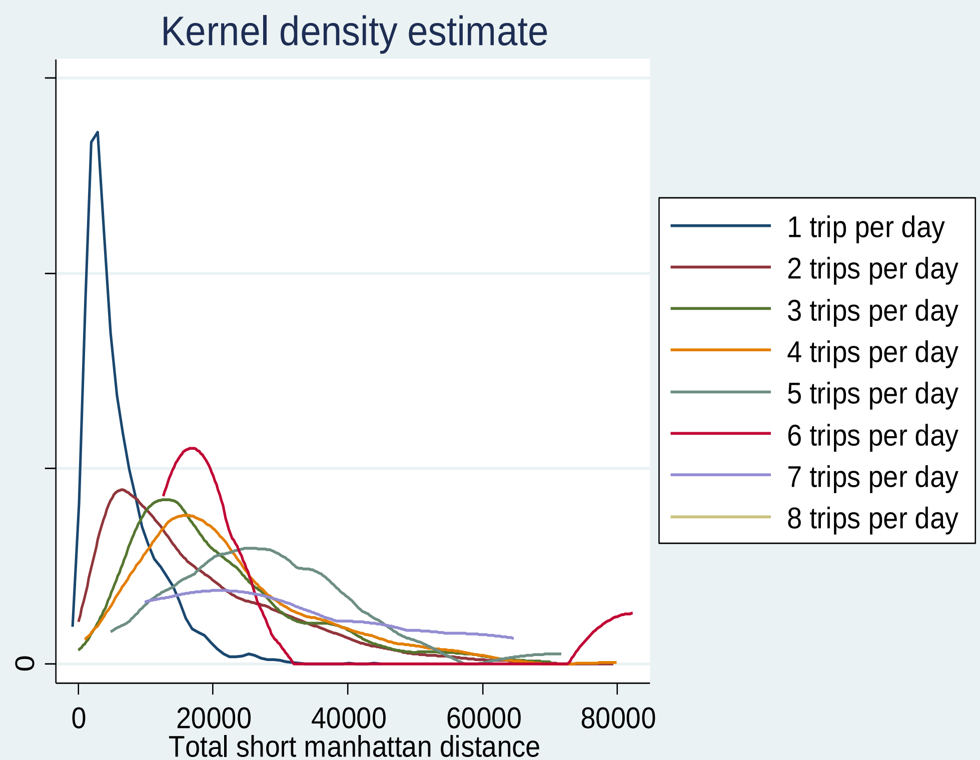




Trip Distance Distribution & Pass ownership



Pass Owners



No Pass Owners

5% Sample of Household Travel

- Total number of trips by transit by each individual
  - Overwhelming number of zero trip makers
- Total distance travelled by transit
  - Zero trip makers have no transit travel distance information
- Transit pass ownership (yes or no)
- Imputed information from secondary sources:
  - Land use and population variables related to place of residences

Further Data Issue

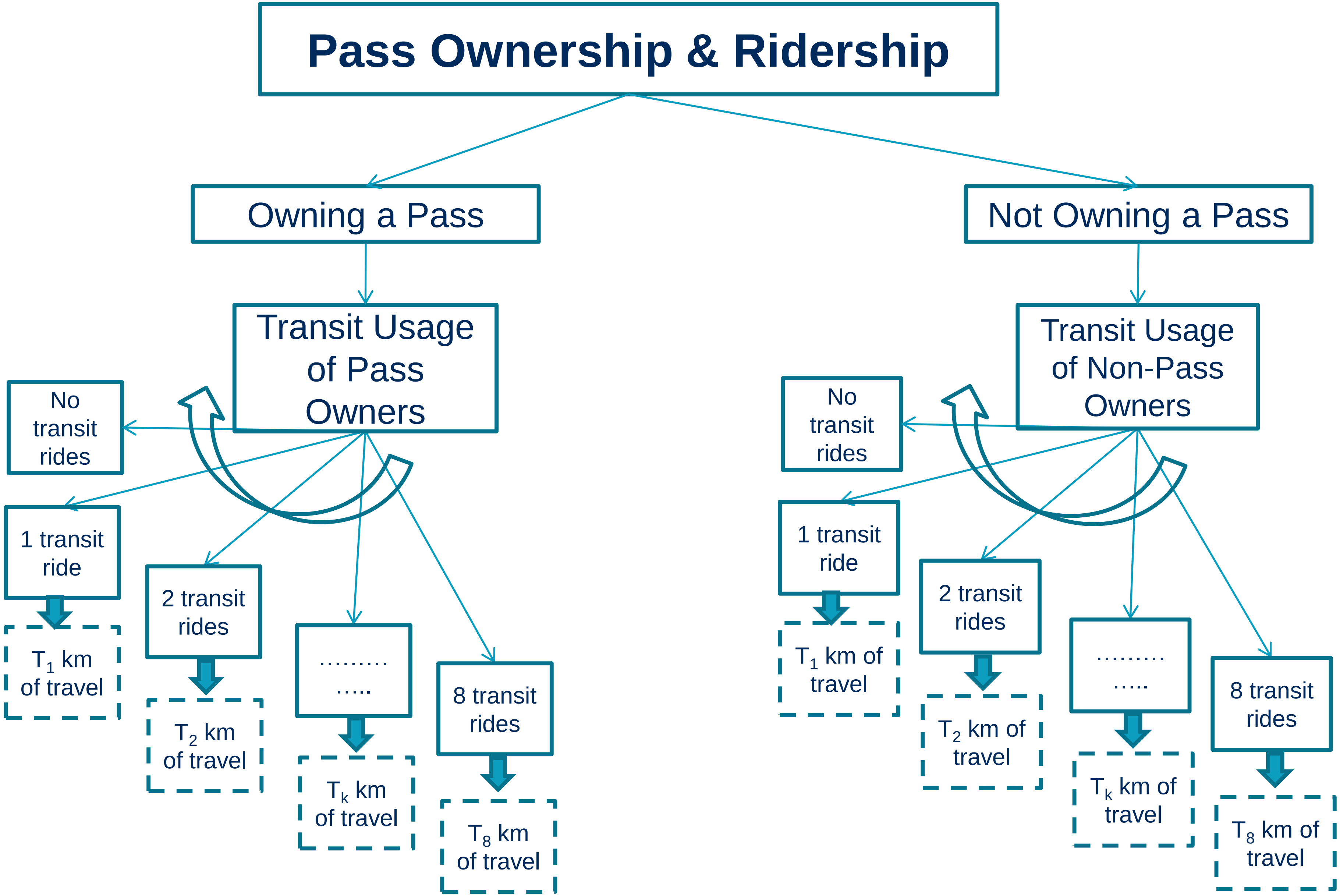
- Difficulty in capturing cost impact?
  - No information on how fare is paid
  - No information on cost subsidy by employers, etc.

- Without precise cost information, we may look at how different factors (that are available in data) explain transit usage

- Such empirical results may have confounded effect of fare/cost, but will allow investigating differences in benefit gain/loss for owning a pass or not

Developing Model that Fits the Context and Data

- How can we evaluate benefit gain from a 1-day travel diary data?
  - Modelling demand for transit usage
  - Develop a modelling structure that allows differentiating patterns of transit usage for transit pass owners and non-transit pass owners
  - Empirically evaluate differences in benefit gain in transit usage for by pass owners and non-pass owners


Pass Ownership & Ridership
Owning a Pass
Not Owning a Pass
Transit Usage of Pass Owners
Transit Usage of Non-Pass Owners
No transit rides
1 transit ride
2 transit rides
……… …..
8 transit rides
T_1 km of travel
T_2 km of travel
T_k km of travel
T_8 km of travel
No transit rides
1 transit ride
2 transit rides
……… …..
8 transit rides
T_1 km of travel
T_2 km of travel
T_k km of travel
T_8 km of travel

Modelling the Demand: Econometrics

- Choice of owning a monthly pass: Binary choice
  - Influenced by expectation of usage of services
  - Other variables?
- Usage of transit services: Number of daily transit trips
  - Count variables with natural ordering
- Total distance travel is correlated with frequency:
  - Continuous variable
- Random Utility Maximization (RUM) Theory:
  - Correlated and nested choice structure
  - Presence of large number of zero trip makers

RUM based Transit Pass Ownership and Usage Model

Utility function of discrete choice of owning/not-owning:

$$U_{pass} = V_{pass} + \varepsilon_{pass} \quad ----------(1)$$

$$U_{no-pass} = V_{no-pass} + \varepsilon_{no-pass} \quad ---------(2)$$

Considering the random components have GEV distribution:

$$\Pr(owning\ \ a\ \ pass) = \exp(V_{pass}) / \left(\exp(V_{pass}) + \exp(V_{no-pass})\right) \quad ------------(3)$$

$$\Pr(not\ owning\ \ a\ \ pass) = \exp(V_{no-pass}) / \left(\exp(V_{pass}) + \exp(V_{no-pass})\right) \quad ---------(4)$$

Further specification of utility of pass ownership:

$$V_{no-pass} = I_{no-pass} \quad ----------(5)$$

$$V_{pass} = I_{pass} + \sum \gamma z \quad ---------(6)$$

$\boldsymbol{I_{no\text{-}pass}}$ is the expected maximum utility of transit usage while not owning a pass

$\boldsymbol{I_{pass}}$ is the expected maximum utility of transit usage while owning a pass

$\sum \gamma z$ is a linear-in-parameter function

RUM based Transit Use Frequency Choices

- Large number of zero usage record:
  - Zero-inflated count variable regression model
  - Zero-inflated Ordered logit/probit regression model
  - RUM based discrete choice mode

- Count variable and ordered regression models do not give a consistent measure of benefit gain from usage of transit services:
  - ✓ Benefit gain would be better measured by expected maximum utility of usages
  - ✓ We need a RUM based discrete choice approach

RUM based Negative Binomial (NB) count choice Model

$$\Pr(y)=\left(\frac{r}{r+\lambda}\right)^{r}\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\lambda}{r+\lambda}\right)^{y}$$

y is the number of trips/usage

r is a non-negative dispersion parameter

λ is the expected number of trips/usage = exp(∑βx)

$$\Pr(y)=\frac{\left(\frac{r}{r+\lambda}\right)^{r}\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\lambda}{r+\lambda}\right)^{y}}{\sum_{y=0}^{Y}\left(\left(\frac{r}{r+\lambda}\right)^{r}\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\lambda}{r+\lambda}\right)^{y}\right)}=\frac{\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\lambda}{r+\lambda}\right)^{y}}{\sum_{y=0}^{Y}\left(\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\lambda}{r+\lambda}\right)^{y}\right)}$$

Denominator is equal to 1

$$\Pr(y)=\frac{\exp(V_y)}{\sum_{y=0}^{Y}\exp(V_y)},\quad \text{Here } V_y=\ln\left(\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\lambda}{r+\lambda}\right)^{y}\right)$$

This is an MNL version of Negative Binomial Choice Model

Add count-specific elements to the systematic utility to capture over/under dispersion of any specific counts

$$\Pr(y)=\frac{\exp(V_y+\eta_y)}{\sum_{y=0}^{Y}\exp(V_y+\eta_y)},\quad \text{Here } \eta_y \text{ is additional count-specific systematic utility}$$

RUM based Negative Binomial (NB)→ Poisson count choice Model

- Negative Binomial (NB) distributing collapses into a Poisson distribution for a large value of r
  - -In case of additional count-specific constants, often r becomes too large to retain the NB formulation

$$\Pr(y) = \frac{e^{-\lambda}\lambda^{y}}{y!}$$

y is the number of trips/usage
r is a non-negative dispersion parameter
λ is the expected number of trips/usage = exp(∑βx)

By using Taylor's series approximation:

$$\Pr(y) = \frac{\lambda^{y}/y!}{\exp(-\lambda)} = \frac{\lambda^{y}/y!}{\sum_{y=0}^{Y}(\lambda^{y}/y!)} = \frac{\exp(\ln(\lambda^{y}/y!))}{\sum_{y=0}^{Y}\exp(\ln(\lambda^{y}/y!))}, \quad \text{Here } V_y = \ln\left(\lambda^{y}/y!\right)$$

This is an MNL version of Poisson Choice Model

Add count-specific elements to the systematic utility to capture over/under dispersion of any specific counts

$$\Pr(y) = \frac{\exp(V_y + \eta_y)}{\sum_{y=0}^{Y}\exp(V_y + \eta_y)}, \quad \text{Here } \eta_y \text{ is additional count-specific systematic utility}$$

Accommodating Ordered nature of Count Choice

- An MNL considers IIA, but alternative counts are ordered!
- A corresponding Ordered Generalized Extreme Value (OGEV) version of the model:

$$\Pr(y) = \frac{e^{\left(\frac{(V_y+\eta_y)}{\rho}\right)}\left[\left(e^{\left(\frac{(V_{y-1}+\eta_{y-1})}{\rho}\right)}+e^{\left(\frac{(V_y+\eta_y)}{\rho}\right)}\right)^{\rho-1}+\left(e^{\left(\frac{(V_y+\eta_y)}{\rho}\right)}+e^{\left(\frac{(V_{y+1}+\eta_{y+1})}{\rho}\right)}\right)^{\rho-1}\right]}{\sum_{r=0}^{R+1}\left(e^{\left(\frac{(V_r+\eta_r)}{\rho}\right)}+e^{\left(\frac{(V_{r-1}+\eta_{r-1})}{\rho}\right)}\right)^{\rho}}$$

$$\text{where} \quad e^{\left(\frac{(V_r+\eta_r)}{\rho}\right)} = 0 \quad \text{for} \quad r<0 \text{ and } r>R$$

Joint Transit Pass Ownership and Use Frequency Choice Model

The joint probability of owning or not owning a transit pass and making **y** number of transit trips in a day: ***P(O-f)***

$$\Pr(owning\ a\ pass\,\&\,making\ y\ transit\ trips\ in\ a\ day) = P(O-f)$$

$$= \left(\frac{ e^{\left(\frac{(Vy+\eta_y)}{\rho}\right)} \left[\left(e^{\left(\frac{(V_y+\eta_y)}{\rho}\right)} + e^{\left(\frac{(V_{y-1}+\eta_{y-1})}{\rho}\right)} \right)^{\rho-1} + \left(e^{\left(\frac{(V_y+\eta y)}{\rho}\right)} + e^{\left(\frac{(V_{y+1}+\eta_{y+1})}{\rho}\right)} \right)^{\rho-1} \right] }{ \left(e^{\left(\frac{(V_0+\eta_0)}{\rho}\right)} \right)^{\rho} + \left(e^{\left(\frac{(V_0+\eta_0)}{\rho}\right)} + e^{\left(\frac{(V_1+\eta_1)}{\rho}\right)} \right)^{\rho} + \ldots\ldots\ldots \left(e^{\left(\frac{(V_{Y-1}+\eta_{Y-1})}{\rho}\right)} + e^{\left(\frac{(V_Y+\eta_Y)}{\rho}\right)} \right)^{\rho} + \left(e^{\left(\frac{(V_Y+\eta_Y)}{\rho}\right)} \right)^{\rho} } \right)$$

$$\times \frac{\exp(I_{pass} + \sum \gamma z)}{\left(\exp(I_{pass} + \sum \gamma z) + \exp(I_{no-pass})\right)}$$

Joint Transit Pass Ownership and Use Frequency Choice Model

- Expected Maximum Utility of Transit Usages:

$$I = \rho \ln \left(\left(e^{\left(\frac{(V_0+\eta_0)}{\rho}\right)} \right)^{\rho} + \left(e^{\left(\frac{(V_0+\eta_0)}{\rho}\right)} + e^{\left(\frac{(V_1+\eta_1)}{\rho}\right)} \right)^{\rho} + \ldots\ldots\ldots \left(e^{\left(\frac{(V_{Y-1}+\eta_{Y-1})}{\rho}\right)} + e^{\left(\frac{(V_Y+\eta_Y)}{\rho}\right)} \right)^{\rho} + \left(e^{\left(\frac{(V_Y+\eta_Y)}{\rho}\right)} \right)^{\rho} \right)$$

- Separate models of usages for pass owners and non-pass owners, results I_{pass} and $I_{no\text{-}pass}$
  - ✓ Separate mean trip rates (λ_1 and λ_2) instead of same rate (λ) for both groups
  - ✓ Separate additional systematic trip rate specific utility functions and/or dispersion parameters (r)

Total Distance Travel Demand

- Considering multiplicative exponential formulations of total demand

$$D=\prod_{k=1}^{K} e^{\alpha_k K} e^{\varepsilon_D}$$

Here, K indicates covariates explaining distance travel

α_k indicates coefficient of covariate k

ε_D is a random variable with normal distribution of zero mean and σ^2 variance.

- As per normal distribution, the probability of observing a total of D km of travel by transit in y trips

$$\Pr(D)=\frac{1}{\sigma}\phi\left(\frac{\ln(D)-\sum_{k=1}^{K}\alpha_k k}{\sigma}\right)$$

Here $\phi(\,.\,)$ the pdf of a standard normal

Transit Pass Ownership and Usage Frequency & Distance Travel Model

- Frequency of transit trips and total distance travelled by transit can be correlated.
- Systematic correlation can be captured by accommodating a same set of explanatory variables in both frequency and distance travel demand model components
- However, correlations among random variables influencing two choices (endogeneity) requires special treatment

Transit Pass Ownership and Usage Frequency & Distance Travel Model

Nested OGEV-Continuous model

$$L_i = \frac{1}{\sigma}\phi\left(\frac{\ln(D)-\sum_{k=1}^{K}\alpha_k k}{\sigma}\right)\Phi_2\left(\Phi^{-1}\left(P(O\text{ - }f)\right)-\tau\left(\frac{\left(\ln(D)-\sum_{k=1}^{K}\alpha_k k\right)/\sigma}{\sqrt{1-\tau^2}}\right)\right)$$

Here $\phi(\,.\,)$ is the pdf of a standard normal

$\Phi^{-1}(\,.\,)$ is the inverse of a univariate standard normal

$\Phi_2(\,.\,)$ is the cdf of a bivariate standard normal

Role of Transit Pass Ownership on Transit Service Demands

Distinguished differences in behaviour of transit pass owners and non-pass owners

- ✓ Separate constant average frequency
- ✓ Differential influences of the factors affecting daily average frequencies of transit usage
- ✓ Differences in overall dispersions of frequencies
- ✓ Differential influences of various factors affecting choices of specific frequencies of transit usages
- ✓ Differences in correlations between unobserved factors affecting transit frequency of usage (as well as pass ownership) and total distance travel demands
- ✓ Differences in variances of total distance travel demands
- ✓ Differential effects of same variables in defining total distance travel demands

Assessing Transit Pass as a Mobility Tool

- A mobility tool would accrue more benefit than just the benefit of daily usage of the tool
  - ✓ Is there any other systematic factors other than benefit drawn from daily usage in transit pass owning?

- Answer to these two would allow testing a hypothesis of transit pass as a mobility tool.

Data for Estimation

Number of individual making no transit trips	101,053
Number of individual making 1 transit trip	3365
Number of individual making 2 transit trips	19191
Number of individual making 3 transit trips	1549
Number of individual making 4 transit trips	868
Number of individual making 5 transit trips	121
Number of individual making 6 transit trips	34
Number of individual making 7 transit trips	11
Number of individual making 8 transit trips	4

Variable	Mean	Standard Deviation
Distance (km) travelled by transit: 1 trip makers	6.57	5.60
Distance (km) travelled by transit: 2 trips makers	19.23	13.80
Distance (km) travelled by transit: 3 trips makers	22.53	13.70
Distance (km) travelled by transit: 4 trips makers	24.75	14.57
Distance (km) travelled by transit: 5 trips makers	29.30	14.64
Distance (km) travelled by transit: 6 trips makers	31.88	18.41
Distance (km) travelled by transit: 7 trips makers	28.10	19.33
Distance (km) travelled by transit: 8 trips makers	25.71	9.61
Distance between home and closest transit stop	0.27	0.16
Distance between home and closest rapid transit station	3.01	2.67
Population density (per sq km) in home zone	7260	6461
Home to Toronto downtown (CBD) distance (km)	14.65	8.05

Data for Estimation

Variables	Sample Proportions (%)
Gender	
Female	44.50
Male	55.47
Driver licence ownership (irrespective of transit trip frequency)	
Yes	63.80
No	36.20
Employment Status	
Full time	29.2
Part time	7.99
Work at home full time	3.86
Work at home part time	1.47
Not employed	57.45
Student Status	
Not a student	84.74
Full time student	13.05
Part time student	2.21
Free Parking at work Place	
Yes	22.79
No	77.21
Home location in the city:	
Planning district 1 (Downtown Toronto)	9
Planning district 2-6 (Inner suburb surrounding Downtown)	39
Planning district 7-9 (Outer suburb on west of inner suburb)	13
Planning district 10-12 (Outer suburb on north of inner suburb)	15
Planning district 13-14 (Outer suburb on east of inner suburb)	11
Planning district 14-16 (Outer suburb on east of inner suburb)	13
Dwelling Type	
House	55.53
Apartment	38.19
Townhouse	6.28

Estimated Model Parameters

Total number of observations	126292	
Loglikelihood of full model	-102225	
Loglikelihood of constant-only model	-273115	
Number of estimated parameters	87	
Rho-Squared value	0.625707	
Variable	Paramete	t-Stat
Average trip rate: Non-pass owners		
Constant	-0.038	-0.25
Female	-0.096	-2.78
Home planning district: 10-12	-0.150	-2.61
Home planning district: 14-16	-0.127	-2.14
Household size	-0.007	-0.42
No driving licese and no household car	0.390	9.66
No driving licese and 1 household car	0.394	6.66
Driving license, but no household car	-0.140	-2.41
Driving license and have car	-0.383	-3.51
Distance between home to CBD	0.055	1.13
Average trip rate: Non-pass owners		
Constant	-0.038	-0.25
Female	-0.069	-1.82
Home planning district: 6-9	-0.158	-2.19
Home planning district: 10-12	-0.050	-0.88
Home planning district: 13-14	-0.077	-1.22
Home planning district: 14-16	-0.142	-1.92
Household size	-0.020	-0.90
No driving licese and no household car	0.384	8.65
No driving licese and 1 household car	0.409	6.82
Driving license, but no household car	0.187	3.60
Driving license and have car	-0.458	-5.44
Distance between home to CBD	0.072	1.33

Additional systematic utility component		
Making 0 trip		
Non-pass owners		
Log of age	1.187	16.24
Total number of cars owned	0.097	7.29
Pass owners		
Log of age	0.851	10.94
Total number of cars owned	-0.259	-17.97
Making 1 trip		
Non-pass owners		
Log of age for age between 17 to 24	-0.328	-4.83
Log of age for age between 24 to 30	-0.259	-4.48
Log of age for age between 30 to 40	-0.310	-5.28
Log of age for age between 40 to 50	-0.230	-3.83
Pass owners		
Log of age for age between 17 to 24	-0.128	-1.36
Log of age for age between 24 to 30	-0.234	-0.59
Log of age for age between 30 to 40	-0.150	-2.20
Log of age for age between 40 to 50	-0.143	-0.86
Making 2 or more trips		
Non-pass owners		
Log of age for age between 17 to 24	-0.038	-0.86
Log of age for age between 24 to 30	-0.137	-1.52
Log of age for age between 30 to 40	-0.053	-1.60
Log of age for age between 40 to 50	-0.052	-1.20
Pass owners		
Log of age for age between 17 to 24	0.389	7.31
Log of age for age between 24 to 30	0.275	4.18
Log of age for age between 30 to 40	0.252	4.23
Log of age for age between 40 to 50	0.292	4.49
Log of age for age between 50 to 60	0.355	5.60
Log of age for age between 60 to 65	0.364	5.49
Log of age for age greater than 65	0.234	3.60

Estimated Model Parameters

Logit Correlation Function for Ordered Trip Rates		
Constant	-0.692	-1.10
Distance between home to CBD	0.260	1.14
Additional (over the daily usage utility) systematic component of pass ownership choice		
Constant	-0.553	-2.37
Distance between home to nearest	-0.011	-2.18
Free parking at work place	-0.732	-20.55
Log of household size	-0.031	-1.08
Student (Full or Part time)	0.516	11.80
Full time worker	1.169	36.26
Part time worker	1.378	31.89
Living in apartments	0.178	6.28
Living in townhouses	0.129	2.54
Log of zonal average after-tax incom	-0.274	-6.07
Contrained Correlation Function for Correlation between unobse factors affecting Trip Rate choice and Total Distance Travel		
Constant for Non-pass owners	4.562	58.39
Constant for Pass owners	4.582	76.51
Variance of Total Distance Travel: Non-pass owners		
1 trip	4.393	28.39
2 trips	4.178	28.78
3 trips	2.393	26.70
4 or more trips	1.949	19.77
Variance of Total Distance Travel: Pass owners		
1 trip	5.112	29.06
2 trips	3.484	35.90
3 trips	1.463	32.07
4 or more trips	1.582	29.51

Total Distance Travel Function: Non-pass owners		
Constants for		
1 trip	-4.710	-24.22
2 trips	-3.321	-16.90
3 trips	-1.442	-7.08
4 trips	-1.099	-5.32
5 trips	-1.322	-4.41
6 trips	-1.572	-2.54
7 or more trips	-1.529	-1.05
Distance between home to CBD	0.289	8.96
Log of age	-0.038	-1.39
Log of zonal average after-tax income in $10(	0.353	9.24
Total Distance Travel Function: Pass owners		
Constants for		
1 trip	-5.329	-28.19
2 trips	-2.129	-13.18
3 trips	-0.333	-2.00
4 trips	-0.380	-2.28
5 trips	-0.290	-1.62
6 trips	-0.233	-1.10
7 or more trips	-0.491	-1.69
Distance between home to nearest transit st	0.030	0.60
Distance between home to CBD	0.459	18.90
Log of age	-0.048	-1.65
Log of zonal average after-tax income in $10(	0.207	7.44

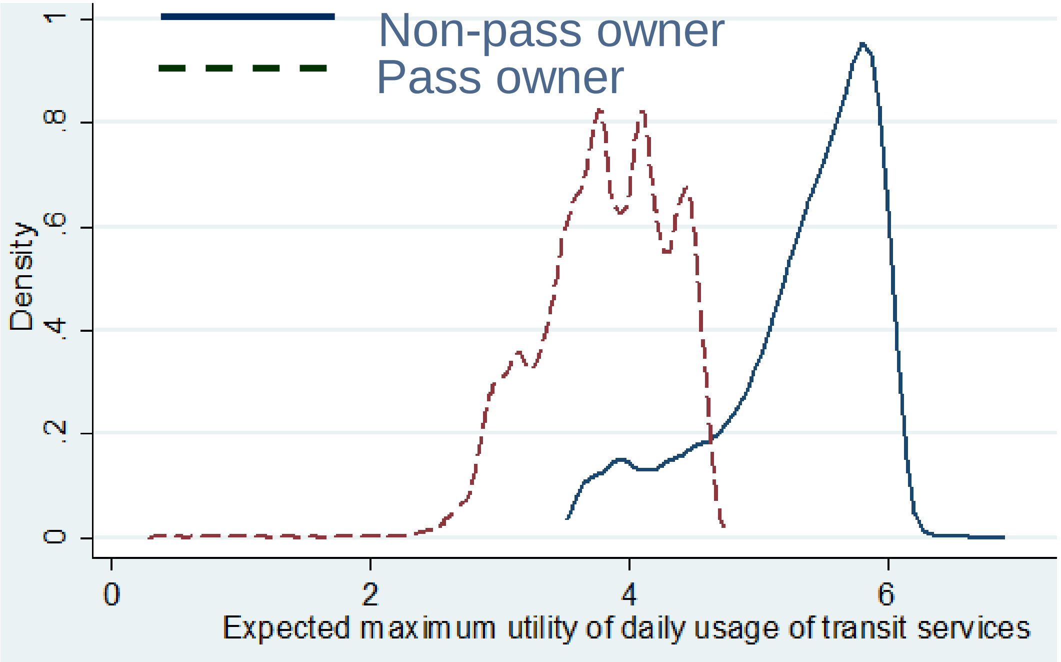
Estimated Model Parameters

Marginal Effect: Transit trip rates of non-pass owners									
	0 trip	*1 trip*	*2 trips*	*3 trips*	*4 trips*	*5 trips*	*6 trips*	*7 trips*	*8 trips*
Female	0	-0.00124	-0.00255	-0.00171	-0.00086	-0.00034	-0.00011	-2.9E-05	-5.1E-05
Home planning district: 10-12	0	-0.00194	-0.00397	-0.00266	-0.00133	-0.00053	-0.00017	-4.6E-05	-7.9E-05
Home planning district: 14-16	0	-0.00164	-0.00336	-0.00225	-0.00113	-0.00044	-0.00014	-3.9E-05	-6.7E-05
Household size	0	-9.1E-05	-0.00019	-0.00012	-6.3E-05	-2.5E-05	-7.9E-06	-2.1E-06	-3.7E-06
No driving licese and no household ca	0	0.005032	0.01031	0.00692	0.003464	0.001366	0.000439	0.000118	0.000206
No driving licese and 1 household car	0	0.005086	0.010421	0.006994	0.003501	0.00138	0.000444	0.00012	0.000209
Driving license, but no household car	0	-0.00181	-0.00371	-0.00249	-0.00125	-0.00049	-0.00016	-4.3E-05	-7.4E-05
Driving license and have car	0	-0.00494	-0.01013	-0.0068	-0.0034	-0.00134	-0.00043	-0.00012	-0.0002
Distance between home to CBD	0	0.000704	0.001443	0.000969	0.000485	0.000191	6.15E-05	1.66E-05	2.89E-05
Marginal Effect: Transit trip rates of pass owners									
	0 trip	*1 trip*	*2 trips*	*3 trips*	*4 trips*	*5 trips*	*6 trips*	*7 trips*	*8 trips*
Female	0	-0.00254	-0.01149	-0.0075	-0.00382	-0.00153	-0.0005	-0.00014	-0.00023
Home planning district: 6-9	0	-0.00583	-0.02633	-0.01718	-0.00874	-0.00351	-0.00115	-0.00031	-0.00054
Home planning district: 10-12	0	-0.00183	-0.00828	-0.0054	-0.00275	-0.0011	-0.00036	-9.9E-05	-0.00017
Home planning district: 13-14	0	-0.00283	-0.01278	-0.00834	-0.00424	-0.0017	-0.00056	-0.00015	-0.00026
Home planning district: 14-16	0	-0.00525	-0.02372	-0.01547	-0.00788	-0.00316	-0.00103	-0.00028	-0.00048
Household size	0	-0.00073	-0.00332	-0.00216	-0.0011	-0.00044	-0.00014	-4E-05	-6.8E-05
No driving licese and no household ca	0	0.014213	0.064234	0.041901	0.021332	0.008554	0.002798	0.000768	0.00131
No driving licese and 1 household car	0	0.015138	0.068412	0.044626	0.022719	0.009111	0.00298	0.000818	0.001396
Driving license, but no household car	0	0.006925	0.031296	0.020415	0.010393	0.004168	0.001363	0.000374	0.000638
Driving license and have car	0	-0.01693	-0.07651	-0.04991	-0.02541	-0.01019	-0.00333	-0.00091	-0.00156
Distance to CBD	0	0.002672	0.012074	0.007876	0.00401	0.001608	0.000526	0.000144	0.000246

Estimated Model Parameters

Non-pass owners		0 trip	1 trip	2 trips	3 trips	4 trips	5 trips	6 trips	7 trips	8 trips
	Log of age	0.012619								
	Total number of cars owned	0.001032								
Pass owners										
	Log of age	0.074418								
	Total number of cars owned	-0.02269								
Non-pass owners										
	Log of age for age between 17 to 24		-0.00423	-0.0005	-0.00022	-8.36E-05	-2.64E-05	-7.06E-06	-1.63E-06	-2.49E-06
	Log of age for age between 24 to 30		-0.00335	-0.00181	-0.00081	-0.0003	-9.57E-05	-2.56E-05	-5.92E-06	-9.04E-06
	Log of age for age between 30 to 40		-0.004	-0.0007	-0.00031	-0.00012	-3.69E-05	-9.89E-06	-2.29E-06	-3.49E-06
	Log of age for age between 40 to 50		-0.00297	-2.15E-08	-1.44E-08	-7.28E-09	-2.89E-09	-9.28E-10	-2.48E-10	-4.30E-10
Pass owners										
	Log of age for age between 17 to 24		-0.00474	0.032519	0.014142	0.0054	0.001732	0.000472	0.000111	0.000166
	Log of age for age between 24 to 30		-0.00865	0.022995	0.01	0.003818	0.001225	0.000334	7.86E-05	0.000117
	Log of age for age between 30 to 40		-0.00556	0.021085	0.009169	0.003501	0.001123	0.000306	7.20E-05	0.000108
	Log of age for age between 40 to 50		-0.0053	0.024396	0.01061	0.004051	0.0013	0.000354	8.34E-05	0.000124
	Log of age for age between 50 to 60			0.029617	0.01288	0.004918	0.001578	0.00043	0.000101	0.000151
	Log of age for age between 60 to 65			0.030388	0.013215	0.005046	0.001619	0.000441	0.000104	0.000155
	Log of age for age greater than 65			0.019524	0.008491	0.003242	0.00104	0.000284	6.67E-05	9.96E-05

Who draws more benefit from daily usage?



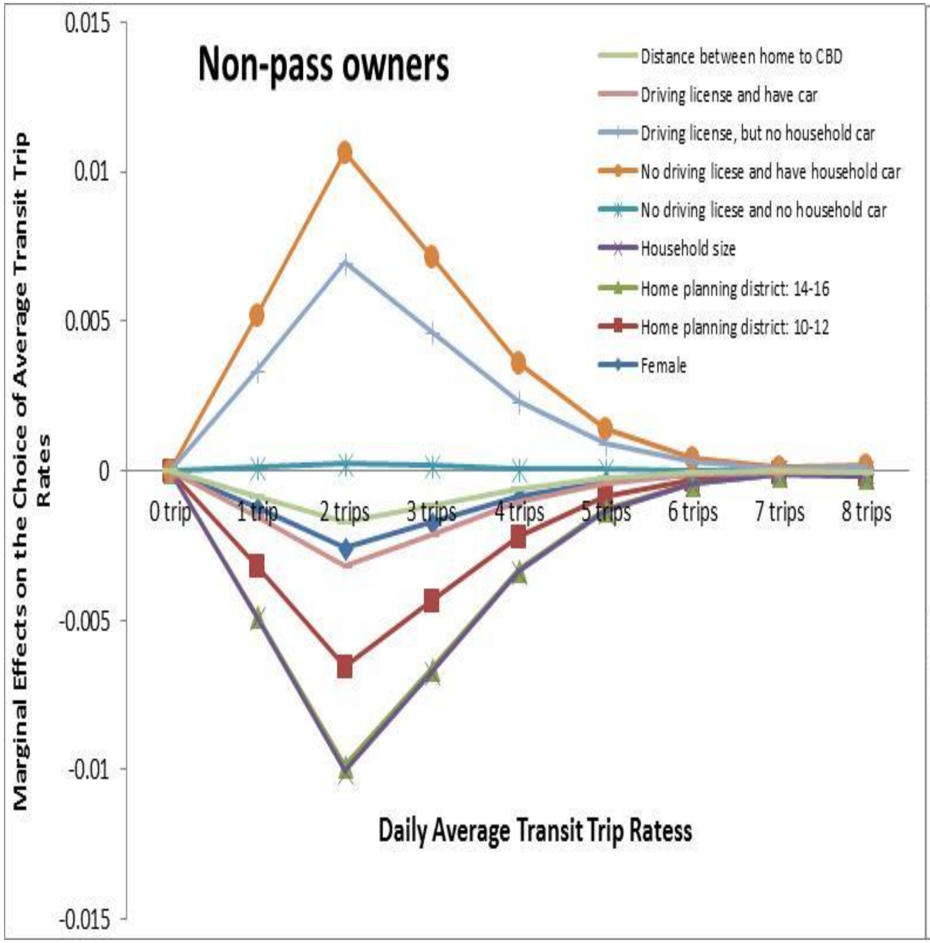
- Non-pass owners seem to have higher consumer surplus/benefit drawn from daily transit usage

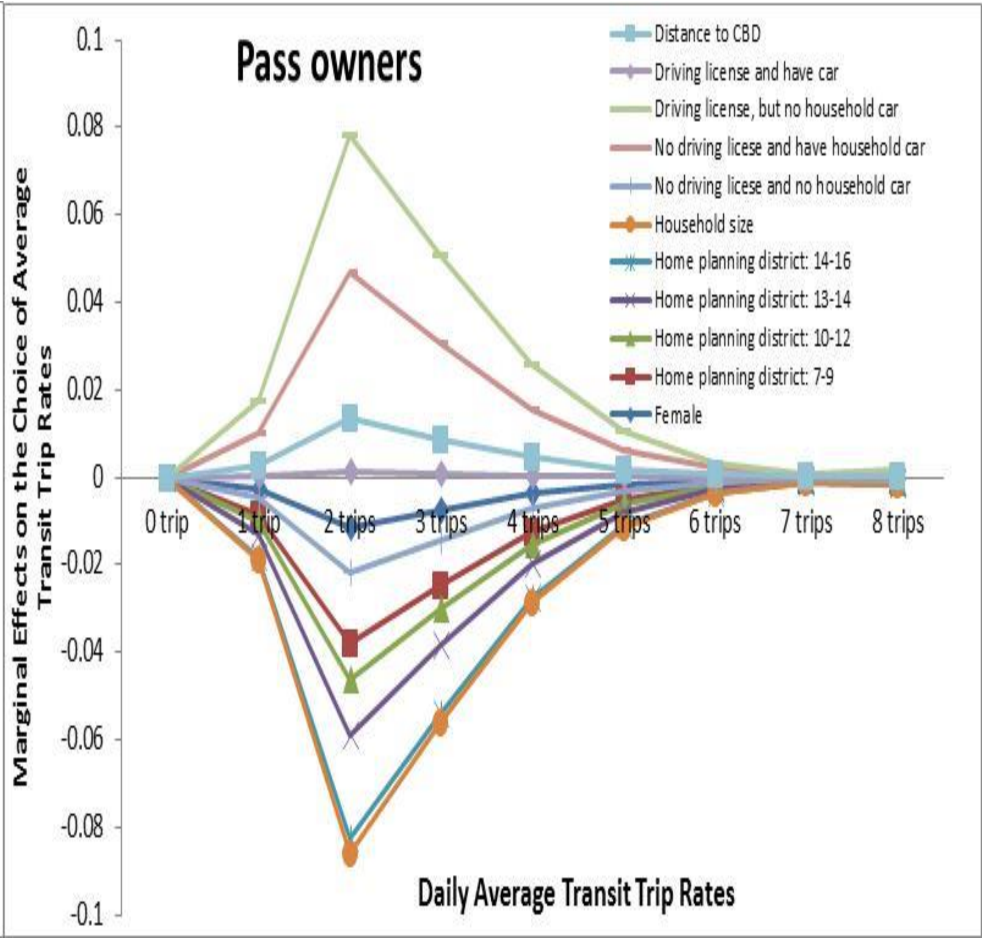
- Pass owners seem to have benefits of owning a pass over and above the benefit drawn from daily usage of the service

What derives the choice of owning a pass?

- Expected maximum utility of daily usage plays great roles
- However, there are moderating factors:
  - ✓ Availability of free parking at work place, longer distance from home to nearest rapid transit station and larger household size tend to lower the attraction of owning a pass
  - ✓ People living in high income zones/neighbourhood have higher positive utility of owning a pass
  - ✓ Having a job (part time or full time) or student status provides positive utility of owning a pass compared to non-workers
  - ✓ Living in apartments or townhouses seem to have positive utility of owning a pass than those living in detached or semi-detached houses

What influences daily frequency?





What influences daily frequency?

- Pass owners are more likely to make more than 1 trip per day if they make any trip

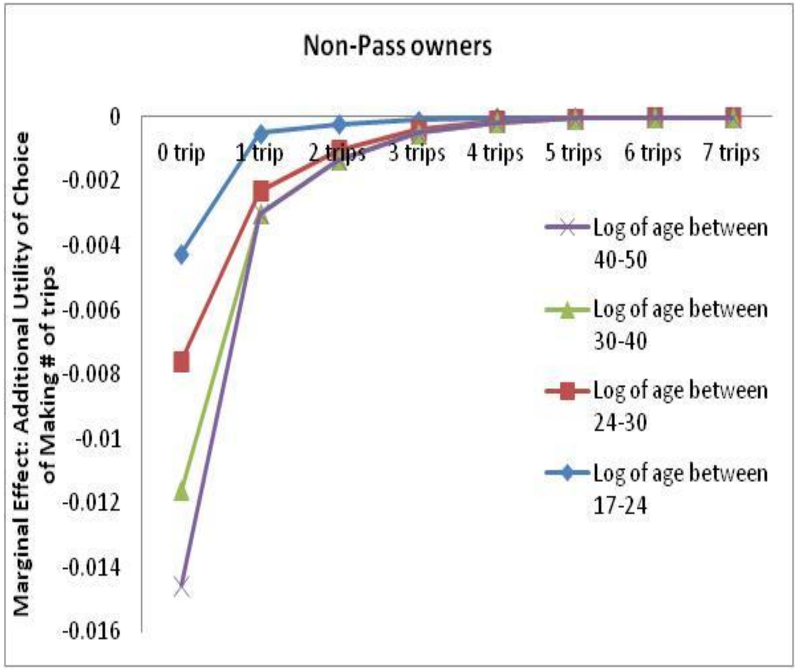
- Marginal effects of the same set of variables on trip frequency are much higher for pass-owners than non-pass owners

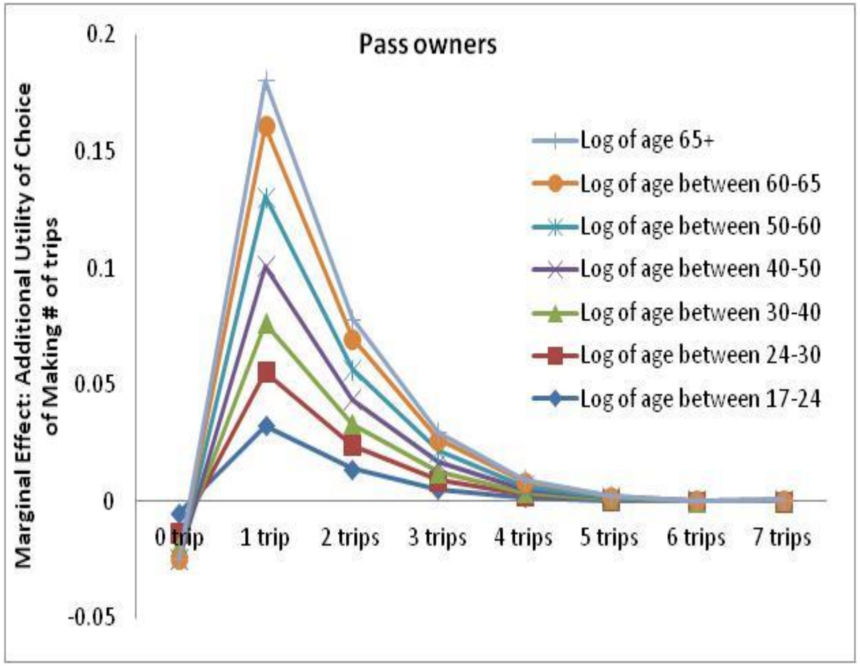
- Spatial variations are clear in differencing effects of different factors on frequency choices of pass and non-pass owners:
    - ✓ People living in western part of outer suburb who don’t own a transit pass do not show any significant pattern of not making such trip

    - ✓ Pass owners living farther from the Downton are more likely to have higher average trip rates, but the opposite is true for non-pass owners

- Overall, females tend to have lower daily trip rate than the males

What influences daily usage? Additional frequency-specific influence





- Age captures individual frequency-specific effects (including zero-inflation for non-pass owners) and show very different for pass and non-pass owners

- Older pass-owners are more likely to make single transit trip

What influences daily distance travel demand?

- Transit station accessibility (distance from home to nearest transit stop) increases the need to travel longer distance by transit
- People living far from the downtown are more likely to travel longer distances
- Older people are less likely to travel longer distance than younger people

Conclusions

- OGEV scale parameter: People living close to the Downtown have stronger correlation between two consecutive trip rates

- Overall contribution of this paper is of two folds: methodological and empirical evidences.
  - ✓ Methodologically: the formulation of a RUM-based count-discrete-continuous model (closed form econometric formulation)

  - ✓ Empirically: evidences of how transit pass ownership can influence different transit usage behaviour

Possible Applications of the Model

- Aggregate transit ridership forecasting → by transit by different groups of people

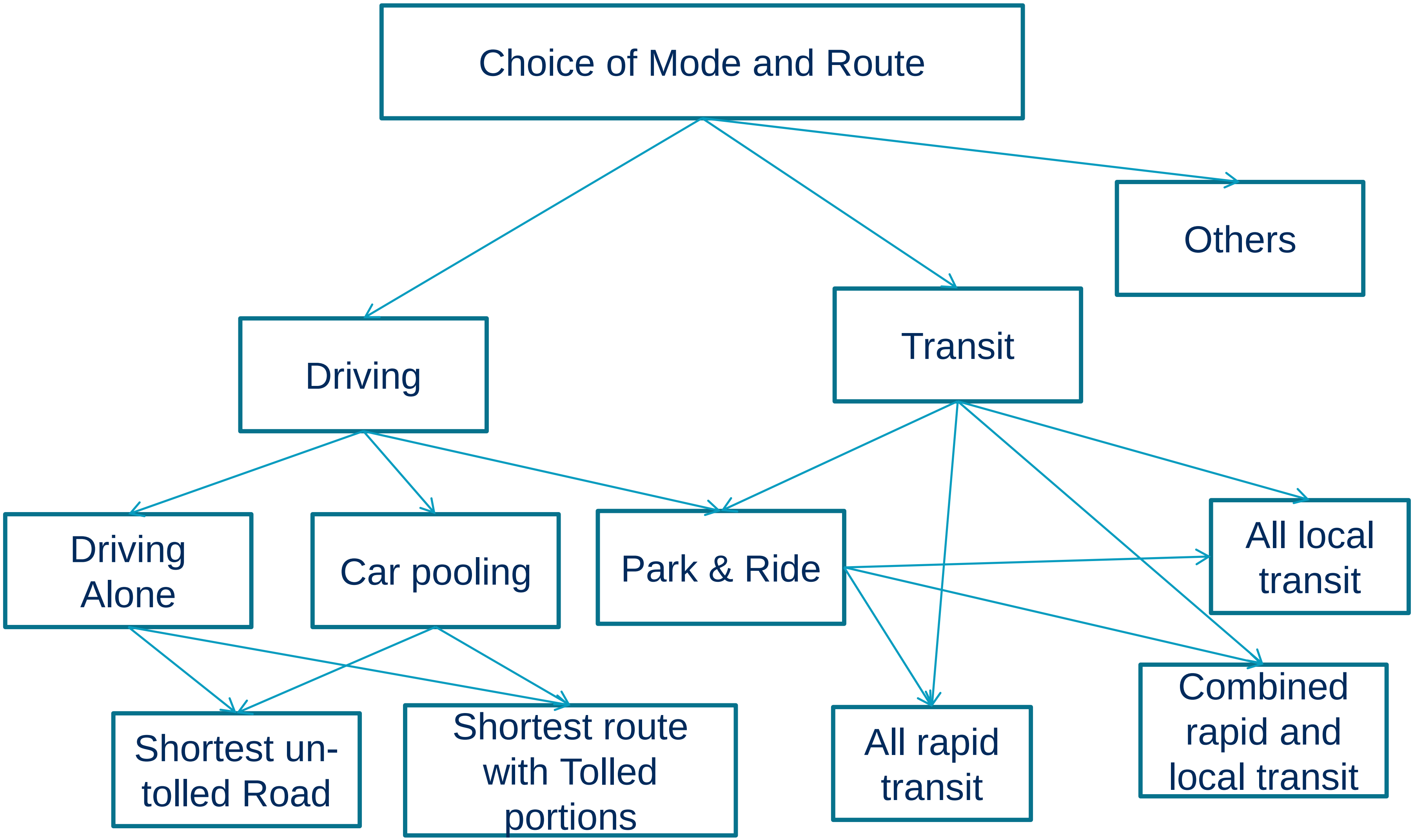
- Assessing impacts of different fare system on different socio-economic group:
  - ✓ Considering revenue neutral assumption, we can estimate optimum rate of distance-based fare

$$(\text{Cost of a retail Ticket}) \times \text{Transit TripFrequency}$$

$$= \text{Base fare (a)} + (\text{Optimum fare per unit distance}) \times \text{Total distance travel}$$

  - ✓ Then using the model to predict total cost incurred to different socio-economic group of people

Related On-going Research: Integrated Model to Evaluate Road Pricing in Multimodal Context



Thank You

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Note: OGEV Formulation

- A GEV formulation:

$$P(k) = \frac{Y_k G_k(.)}{G(....)}$$

- K means k[th] alternative
- G(.) is the GEV generating function
- $G_k(.)$ is partial derivative of G(.) with respect to k[th] argument of G(.)

- Consider a GEV Function:

$$G(....) = G(Y_1, Y_2, ...Y_k, ...Y_J)$$

- For an Ordered GEV Function

$$G(....) = \left(\cancelto{0}{\frac{1}{2} Y_0^{\frac{1}{1-\sigma}}} + \frac{1}{2} Y_1^{\frac{1}{1-\sigma}} \right)^{(1-\sigma)} + \left(\frac{1}{2} Y_1^{\frac{1}{1-\sigma}} + \frac{1}{2} Y_2^{\frac{1}{1-\sigma}} \right)^{(1-\sigma)} + ...$$

$$.......................\left(\frac{1}{2} Y_{J-1}^{\frac{1}{1-\sigma}} + \frac{1}{2} Y_J^{\frac{1}{1-\sigma}} \right)^{(1-\sigma)} + \left(\frac{1}{2} Y_J^{\frac{1}{1-\sigma}} + \cancelto{0}{\frac{1}{2} Y_{J+1}^{\frac{1}{1-\sigma}}} \right)^{(1-\sigma)}$$

Note: OGEV Formulation

- Probability of kth alternative: Considering ρ=1-σ

$$P(k)=\frac{\left(e^{V_k/\rho}\right)\left(\left(e^{V_{k-1}/\rho}+e^{V_k/\rho}\right)^{\rho-1}+\left(e^{V_k/\rho}+e^{V_{k+1}/\rho}\right)^{\rho-1}\right)}{\sum_{j=1}^{J+1}\left(e^{V_{j-1}/\rho}+e^{V_k/\rho}\right)^{\rho}}$$

$$P(k)=\frac{\left(e^{V_k/\rho}\right)\left(e^{V_{k-1}/\rho}+e^{V_k/\rho}\right)^{\rho-1}}{\sum_{j=1}^{J+1}\left(e^{V_{j-1}/\rho}+e^{V_k/\rho}\right)^{\rho}}+\frac{\left(e^{V_k/\rho}\right)\left(e^{V_k/\rho}+e^{V_{k+1}/\rho}\right)^{\rho-1}}{\sum_{j=1}^{J+1}\left(e^{V_{j-1}/\rho}+e^{V_k/\rho}\right)^{\rho}}$$

$$P(k)=\frac{\left(e^{V_k/\rho}\right)}{\left(e^{V_{k-1}/\rho}+e^{V_k/\rho}\right)^{1}}\frac{\left(e^{V_{k-1}/\rho}+e^{V_k/\rho}\right)^{\rho}}{\sum_{j=1}^{J+1}\left(e^{V_{j-1}/\rho}+e^{V_j/\rho}\right)^{\rho}}+\frac{\left(e^{V_k/\rho}\right)}{\left(e^{V_k/\rho}+e^{V_{k+1}/\rho}\right)^{1}}\frac{\left(e^{V_k/\rho}+e^{V_{k+1}/\rho}\right)^{\rho}}{\sum_{j=1}^{J+1}\left(e^{V_{j-1}/\rho}+e^{V_j/\rho}\right)^{\rho}}$$

Note: OGEV Alternative Formulation

- Probability of k^{th} alternative: Considering $\rho=1-\sigma$

$$P(k)=\frac{\left(e^{V_k/\rho}\right)}{\left(e^{V_{k-1}/\rho}+e^{V_k/\rho}\right)^1}\frac{\left(e^{V_{k-1}/\rho}+e^{V_k/\rho}\right)^\rho}{\sum_{j=1}^{J+1}\left(e^{V_{j-1}/\rho}+e^{V_k/\rho}\right)^\rho}+\frac{\left(e^{V_k/\rho}\right)}{\left(e^{V_k/\rho}+e^{V_{k+1}/\rho}\right)^1}\frac{\left(e^{V_k/\rho}+e^{V_{k+1}/\rho}\right)^\rho}{\sum_{j=1}^{J+1}\left(e^{V_{j-1}/\rho}+e^{V_k/\rho}\right)^\rho}$$

$$P(k)=\frac{\left(e^{V_k/\rho}\right)}{\left(e^{V_{k-1}/\rho}+e^{V_k/\rho}\right)}\frac{\exp\left(\rho\ln\left(e^{V_{k-1}/\rho}+e^{V_k/\rho}\right)\right)}{\sum_{j=1}^{J+1}\exp\left(\rho\ln\left(e^{V_{j-1}/\rho}+e^{V_j/\rho}\right)\right)}$$

$$+\frac{\left(e^{V_k/\rho}\right)}{\left(e^{V_k/\rho}+e^{V_{k+1}/\rho}\right)}\frac{\exp\left(\rho\ln\left(e^{V_k/\rho}+e^{V_{k+1}/\rho}\right)\right)}{\sum_{j=1}^{J+1}\exp\left(\rho\ln\left(e^{V_{j-1}/\rho}+e^{V_j/\rho}\right)\right)}$$

$$P(k)=P(k\,|\,k-1,k)P(k-1,k)+P(k\,|\,k,k+1)P(k,k+1)$$

Note: OGEV Alternative Formulation

$$\text{Marginal Effect} = \sum_{O} \Pr(O)\Pr(k \mid O)\left[(1-\Pr(k)) + \left(\frac{1}{\rho}-1\right)(1-\Pr(k \mid O))\right]\left(\frac{\partial V_k}{\partial x}\right)$$

here, O indicates the clusters of pairs

$$= \left[\begin{array}{l} \Pr(k-1,k)\Pr(k \mid k-1,k)\left[(1-\Pr(k)) + \left(\frac{1}{\rho}-1\right)(1-\Pr(k \mid k-1,k))\right] \\ + \Pr(k,k+1)\Pr(k \mid k,k+1)\left[(1-\Pr(k)) + \left(\frac{1}{\rho}-1\right)(1-\Pr(k \mid k,k+1))\right] \end{array}\right]\left(\frac{\partial V_k}{\partial x}\right)$$

Note: OGEV Alternative Formulation

$$\text{Direct Elasticity} = \sum_{O} \Pr(O)\Pr(k\,|\,O)\frac{\left[(1-\Pr(k)) + \left(\frac{1}{\rho}-1\right)(1-\Pr(k\,|\,O))\right]}{\Pr(k)}\left(\frac{\partial V_k}{\partial x}\right)x$$

here, O indicates the clusters of pairs

$$= \left[\begin{array}{l} \Pr(k-1,k)\Pr(k\,|\,k-1,k)\dfrac{\left[(1-\Pr(k)) + \left(\frac{1}{\rho}-1\right)(1-\Pr(k\,|\,k-1,k))\right]}{\Pr(k)} \\ + \Pr(k,k+1)\Pr(k\,|\,k,k+1)\dfrac{\left[(1-\Pr(k)) + \left(\frac{1}{\rho}-1\right)(1-\Pr(k\,|\,k,k+1))\right]}{\Pr(k)} \end{array}\right]\left(\frac{\partial V_k}{\partial x}\right)x$$

$$\text{If the alternative are not paired : Indirect Elasticity} = \text{-}\Pr(\mathrm{k})\left(\frac{\partial V_k}{\partial x}\right)x$$

$$\text{If the alternative is paired : Indirect Elascticity } = \text{-}\left[\Pr(\mathrm{k}) + \frac{\left(\frac{1}{\rho}-1\right)\Pr(k,k-1)\Pr(k\,|\,k,k-1)\Pr(k-1\,|\,k,k-1)}{\Pr(\mathrm{k-1})}\right]\left(\frac{\partial V_{k-1}}{\partial x}\right)x$$

$$\text{OR} \qquad == \text{-}\left[\Pr(\mathrm{k}) + \frac{\left(\frac{1}{\rho}-1\right)\Pr(k,k+1)\Pr(k\,|\,k,k+1)\Pr(k+1\,|\,k,k+1)}{\Pr(\mathrm{k+1})}\right]\left(\frac{\partial V_{k+1}}{\partial x}\right)$$

Note: OGEV Alternative Formulation

- For Negative Binomial formulation

$$V_k = V_y = \ln\left(\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\lambda}{r+\lambda}\right)^y\right) + \beta' z = \ln\left(\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\exp(\beta x)}{r+\exp(\beta x)}\right)^y\right) + \beta' z$$

$$So,\quad \frac{\partial V_y}{\partial x} = \left(\frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r}\left(\frac{\exp(\beta x)}{r+\exp(\beta x)}\right)^y\right)^{-1} \times \frac{\Gamma(r+y)}{\Gamma(y+1)\Gamma r} \times y\left(\frac{\exp(\beta x)}{r+\exp(\beta x)}\right)^{y-1}$$

$$\times \frac{\left(r+\exp(\beta x)\right)\exp(\beta x)\beta - \exp(\beta x)\exp(\beta x)\beta}{\left(r+\exp(\beta x)\right)^2}$$

$$= y\beta\left(1 - \frac{\exp(\beta x)}{r+\exp(\beta x)}\right) = y\beta\lambda\left(\frac{r}{r+\lambda}\right)$$

$$\frac{\partial V_y}{\partial z} = \beta'$$

- For Poisson formulation

$$V_y = \ln\left(\lambda^y / y!\right) + \beta' z = \ln(\exp(\beta x))^y - \ln(y!) + \beta' z = y(\beta x) - \ln(y!) + \beta' z$$

$$So,\quad \frac{\partial V_y}{\partial x} = y\beta$$

$$\frac{\partial V_y}{\partial z} = \beta'$$