Evidence-Based Transportation Demand Analysis

Aggregate Demand Modelling

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Outline

- Approaches of Travel Demand Modelling
- Aggregate Demand Models:
  - Total Demand Models
  - Issues with Total Demand Modelling
  - Modelling Shares of Alternative Demands
  - Modelling Total Demand along with Shares
- Ad-hoc Modelling approaches:
  - Elasticity-based Model
  - Pivot-point Updating

Travel Demands

- Urban travel demands
  - Number of trips per day by individuals/households/traffic zones
  - Proportions of trips by different modes: Transit ridership
  - Proportion of trips on different routes/links/corridors: Transit lines
- Inter-city travel demands:
  - Total trips by different modes (bus, rail, air, car) between cities
- International travel demands:
  - Total number of passenger travel within region, inter-region, between countries, between continents
  - Passenger arrival rates by ground transpiration modes (bus, rail, car)
- Tourism travel demands / Special travel generators:
  - Trips generated by hotels
  - Trips attracted by hospitals
  - Trips attracted to historical sites, recreational locations, etc.

Approaches of Travel Demand Models

- Based on the concept of utility:
  - Direct utility: when utility function is a function of quantity demands
  - Indirect utility: when utility function is not a function of quantity
- Based on uncertainty in predictions:
  - Deterministic method: trip rate tables, diversion curves etc.
  - Stochastic method: application of stochastic econometric models
- Based on mathematical optimization approach:
  - Random utility maximization
  - Fitting linear/non-linear demand curves
- Based on level of aggregate of travel demands:
  - Aggregate demand model: modelling aggregation of choices
  - Disaggregate demand model: modelling individual choices

Aggregate versus Disaggregate Model

- Choice of aggregate versus disaggregate demand models:
  - Purpose and context of study
  - Data availability and resource available for model development
  - Disaggregate models are always better than aggregate models if and only if data are available and computational burdens are allowed
- In practice, aggregate models complements disaggregate models
- Aggregate demand models:
  - Suppresses heterogeneity in travel demand
  - Suppresses variability in travel demands
  - Future predictions are uncertain future when extrapolation is problematic

Aggregate Demand Models

- Aggregate Demand Models
  - Total Demand Models
    - Linear Model
    - Log-linear /Cobb-Douglas Form Model
    - Box-Cox regression
  - Market Segmentation Model
    - Linear Logit Model
  - Total Demands & Market Segmentation
    - Translog Demand System Model
    - Constant Elasticity of Substitution (CES) Demand System Model
    - Almost Ideal Demand System (AIDS) Model

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Linear Model

- Linear regression model:

$$\text{Total Demand, D (e.g. number of trips)} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots\dots\dots + \beta_N x_n + \varepsilon = \beta_0 + \sum_i \beta_1 x_1 + \varepsilon$$

$$\Pr(D) = f(\varepsilon) = \text{the } pdf \text{ of } \varepsilon \text{ ; consiering that the mean value of } \varepsilon \text{ is} \left(\beta_0 + \sum_i \beta_i x_i \right)$$

- Elasticity of demand (function of x_j)

$$E = \frac{\partial D/D}{\partial x_j / x} = \frac{\partial D}{\partial x_j} \times \frac{x_j}{D} = \frac{\beta_j}{D} x_j$$

- Model estimations: Least-Square or Maximum Likelihood
- Limitations:
  - Assumption of linear functional form
  - No restriction on ‘zero value’ prediction
  - Difficult to handle data with large portion of ‘zero values’ of dependent variables

Log-Linear / Cobb-Douglas Form Model

- Multiplicative functional form:

$$\text{Total Demand, D (e.g. number of trips)} = e^{\beta_0} x_1^{\beta_1} x_2^{\beta_2} \ldots e^{\beta_k x_k} \ldots\ldots x_n^{\beta_N} \; e^{\varepsilon}$$

$$\ln(\mathrm{D}) = \beta_0 + \beta_1 \ln(x_1) + \beta_2 \ln(x_2) + \ldots + \beta_k x_k + \ldots .\beta_n \ln(x_n) + \varepsilon$$

$$\Pr(\mathrm{D}) = \mathrm{f}(\varepsilon) = \text{the } \textit{pdf} \text{ of } \varepsilon \text{ ; consiering that the mean value of } \varepsilon \text{ is} \left(\beta_0 + \sum_i \beta_i \log(x_i) \right)$$

- Elasticity of demand

$$E = \frac{\partial D}{\partial x_j} \frac{x_j}{\mathrm{D}} = \beta_j$$

- Model estimations: Least-Square or Maximum Likelihood
- Limitations:
  - Fixed elasticity
  - Handling 'zero value' in the observed data needs extra care

Log-Linear / Cobb-Douglas Form Model

- Multiplicative functional form gives flexibility of various possible forms (easy to handle categorical/dummy variable)

$$D = x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta 3} .. x_n^{\beta_n} ... \exp(\beta_0 + \gamma_1 y_1 + ... + \gamma_n y_n + \varepsilon)$$

$$= \prod_{i=1}^{n} x_i^{\beta_i} \prod_{j=1}^{m} \exp(\beta_0 + \gamma_j y_j)$$

$$\ln(D) = \beta_0 + \sum_{i=1}^{n} \beta_i \ln(x_i) + \sum_{j=1}^{n} \gamma_j y_j + \varepsilon$$

Box-Cox Regression Model

- Non-linear transformation of dependent and/or independent variables

$$\text{Total Demand, D}(\lambda) = \sum_j \beta x_j(\lambda) + \sum_j \beta z_j + \varepsilon$$

$$D(\lambda) = D^\lambda = \begin{cases} \dfrac{D^\lambda - 1}{\lambda} & for \quad \lambda \neq 0 \\ \ln(D) & for \ \lambda = 0 \end{cases} \qquad x(\lambda) = x^\lambda = \begin{cases} \dfrac{x^\lambda - 1}{\lambda} & for \quad \lambda \neq 0 \\ \ln(x) & for \ \lambda = 0 \end{cases}$$

- Elasticity of demand

$$E = \frac{\partial D}{\partial x_j}\frac{x_\mathrm{j}}{D} = \beta_\mathrm{j}\left(\frac{D}{x_j}\right)^\lambda$$

- Model estimations: Least-Square or Maximum Likelihood

Handling Zero Demand

- Presence of 'zero value' of observed demand in the dataset causes problem in model estimation as well as prediction
- Ad-hoc way of handling 'zero value':
  - For Cob-Douglas formulation: offset all observation by 1 so that for the 'zero value' it becomes log(D+1)=log(0+1)=0
  - For linear regression cannot handle 'zero demand'. So, one has to remove data of 'zero demand'
- In case, the 'zero demands' are legitimate and has a reasonable share in the observed data, a sample selection approach needs to be taken:

$$\text{Total Demand, D} = \left(\Pr(D=0) + \Pr(D \neq 0) \times \Pr(D)\right)$$

$$\Pr(D=0) = \Phi(\gamma_o + \gamma_1 z_1 + \ldots)$$

$$\Pr(D \neq 0) = 1 - \Phi(\gamma_o + \gamma_1 z_1 + \ldots)$$

$$\text{Total Demand, D} = \left(\Phi(\gamma_o + \gamma_1 z_1 + \ldots) + (1 - \Phi(\gamma_o + \gamma_1 z_1 + \ldots)) \times \Pr(D)\right)$$

Lag effect in Aggregate Demand Modelling

- Lag effects:
  - Lag effect of changes in supply (cost/price), context (socio-economic)
  - Lag effect of changes in total demand

- Aggregate demand model can handle lag effect:
  - Total observed demand, D is the equilibrium demand
  - Actual demand or Desired Demand, D^* is unobserved
  - Assumption is that demand, the observed or equilibrium demand (D) adjusts partially towards desired level over the time steps of analysis (t):

$$\ln(D_t) - \ln(D_{t-1}) = \mu(\ln(D_t^*) - \ln(D_{t-1}))$$

$$\mu \textit{ is the rate of adjustment and } 0 < \mu < 1$$

- ✓ Higher the value of μ quicker is the adjustment.
- ✓ Instant adjustment if the value of $\mu = 1$
- ✓ No adjustment, if $\mu = 0$

Lag effect in Aggregate Demand Modelling

- Since actual demand or desired demand is unobserved, we can consider it as a random variable

$$\text{Latent unobserved desired demand, } \mathrm{D}_{\mathrm{t}}^{*} = \prod_{i=1}^{n} x_i^{\beta_i} \times \prod_{j=1}^{m} \exp(\beta_0 + \gamma_j y_j) \times \exp(\varepsilon)$$

$$\text{Taking logarithm}: \ln(\mathrm{D}_{\mathrm{t}}^{*}) = \beta_0 + \sum_{i=1}^{n} \ln(x_i^{\beta_i}) + \sum_{j=1}^{m} \gamma_j y_j + \varepsilon \quad ....(1)$$

$$\textit{Considering lagged demand effects,}\ \ln(D_t) - \ln(D_{t-1}) = \mu(\ln(D_t^{*}) - \ln(D_{t-1})) \quad \textit{where}\ \text{and}\ 0 < \mu < 1$$

$$=> \mu \ln(D_t^{*}) = (\mu - 1)\ln(D_{t-1}) + \ln(D_t) \qquad => \ln(D_t^{*}) = (1 - 1/\mu)\ln(D_{t-1}) + \ln(D_t)/\mu$$

$$\text{Replacing } D_t^{*} \text{ in orginal demand function (1)}: (1 - 1/\mu)\ln(D_{t-1}) + \ln(D_t)/\mu = \beta_0 + \sum_{i=1}^{n} \beta_i \ln(x_i) + \sum_{j=1}^{m} \gamma_j y_j + \varepsilon$$

$$\ln(D_t) = (1 - \mu)\ln(D_{t-1}) + \mu(\beta_0 + \sum_{i=1}^{n} \beta_i \ln(x_i) + \sum_{j=1}^{m} \gamma_j y_j + \varepsilon)$$

$$\text{Final Demand Model}: \ln(D_t) = \mu' \ln(D_{t-1}) + \beta_0' + \sum_{i=1}^{n} \beta_i' \ln(x_i) + \sum_{j=1}^{m} \gamma_j' y_j + \varepsilon'$$

Lag effect in Aggregate Demand Modelling

Aggregate Model with lagged demand

$$\ln(D_t) = \mu^{/} \ln(D_{t-1}) + \beta_0^{/} + \sum_{i=1}^{n} \beta_i^{/} \ln(x_i) + \sum_{j=1}^{m} \gamma_j^{/} y_j + \varepsilon^{/}$$

Aggregate Model without lagged demand

$$\ln(D_t) = \beta_0 + \sum_{i=1}^{n} \beta_i \ln(x_i) + \sum_{j=1}^{m} \gamma_j y_j + \varepsilon$$

- Lagged effect model can be estimated a linear regression model
- Model estimated without considering lagged demand will give static elasticity
- Model estimated with lagged demand effect will give long-term elasticity

Example: Modelling Demand of Inter-City Train

Cobb-Douglas Specification:

$$J_t = \beta_0 \; J_{t-1}^{\beta_1} G_t^{\beta_2} F_t^{\beta_3} T_t^{\beta_4} \exp\left(\beta_5 t + \sum_{k=1}^{K} \gamma_k x_{kt} \right)$$

- J_t is the total demand (passenger-miles) at time t
- J_{t-1} is the total demand (passenger-miles) at time t-1
- G_t is the income variable at time t
- F_t is the rail fare price variable
- T_t is a rail performance measure
- t is the linear time trend
- x_k is a vector of other variables, including dummies and variables relating to competing modes
- β_0 , β_1 , β_2 , β_3 , β_4 , β_5 , γ_k etc. are the parameters of the model (to be estimated)

Example: Modelling Demand of Inter-City Train

- Observed dataset does not have any 'zero value': Used log-normal

$$Ln(J_t) = \ln(\beta_0) + \beta_1 \ln(J_{t-1}) + \beta_2 \ln(G_t) + \beta_3 \ln(F_t) + \beta_4 \ln(T_t) \\ + \left(\beta_5 t + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + \gamma_4 x_4 + \ldots\ldots\ldots\right)$$

$$J_t = \exp\begin{pmatrix} \ln(\beta_0) + \beta_1 \ln(J_{t-1}) + \beta_2 \ln(G_t) + \beta_3 \ln(F_t) + \beta_4 \ln(T_t) \\ + \beta_5 t + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3 + \gamma_4 x_4 + \ldots\ldots\ldots \end{pmatrix}$$

Example: Modelling Demand of Inter-City Train

Variable selection: Based on data availability

- It was considered essential to include variables covering a number of market and service characteristics:
  - Demand lag, and time trend
  - Own-price elasticity
  - Cross-elasticity to auto and air, especially fares
  - Service quality in terms of on-time performance
  - Macroeconomic factors (GDP, income)
  - Seasonality

Example: Modelling Demand of Inter-City Train

Variable	Period	Form
Lag demand: Ln of Pax-mile	Quarter	Lag (years) = 1 / (1-parameter)
Rail Fare: Rev. per pax-mi	Quarter	Coefficient > 1 : *elastic*
On-Time Performance (OTP)	Quarter	Ln (%)
Average Employment	Quarter	Ln (Millions)
Income Proxy (Avg. Market's Emplmt. x GDP / Employed)	Quarter	Ln ($ Millions)
Time Trend	Quarter	units
Quarterly Dummies	Quarter	0 or 1
Gasoline Price	Quarter	Deflated (real) price
Airfare	Annual	Deflated (real) price
GDP of European Union	Quarter	Real GDP

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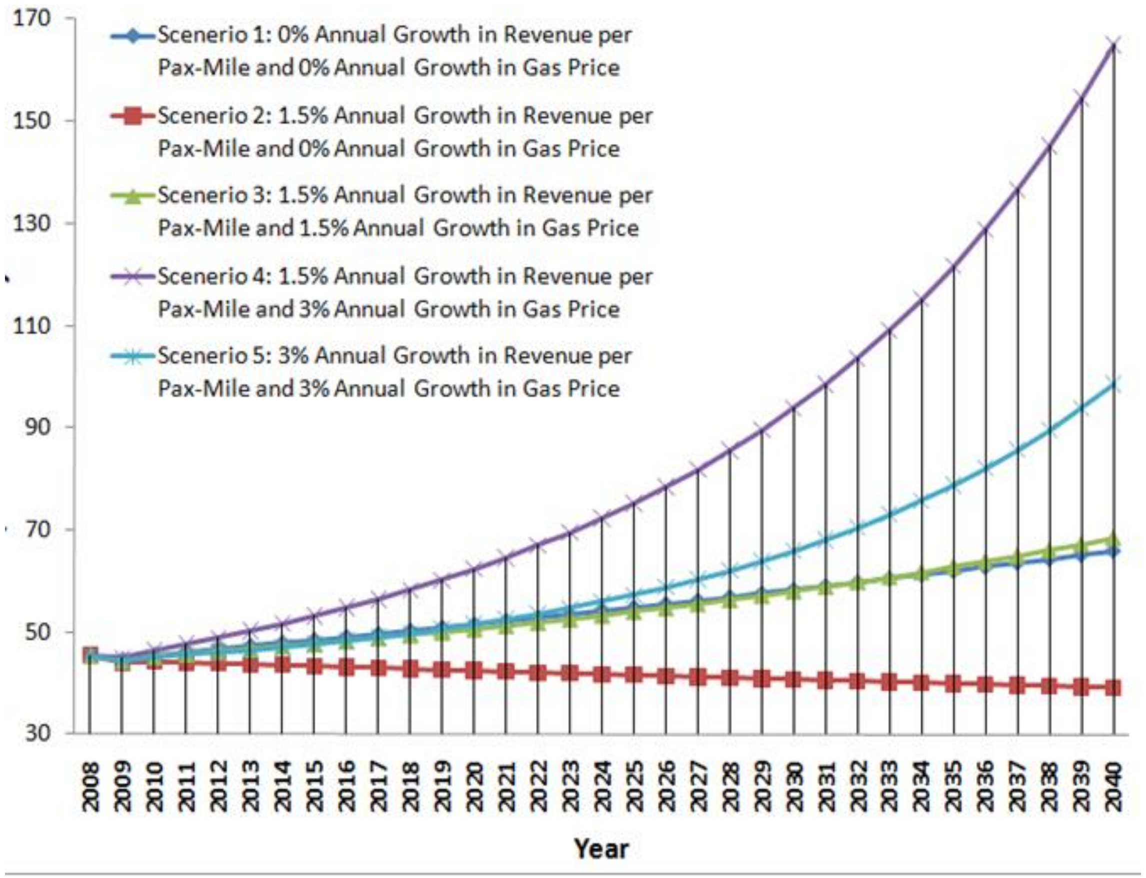
Example: Modelling Demand of Inter-City Train

R-squared = 0.93 *Variables*	*Coef.*	*Robust Std. Err.*	*t-Stat*
Total Pax-Miles in Previous Quarter	0.518	0.154	3.360
Avg Revenue per Pax-Mile	-0.533	0.288	-1.850
OTP	0.132	0.100	1.320
Avg Employment in Millions	2.045	0.841	2.430
Time Step	-0.029	0.019	-1.490
Quarter 1 Dummy	0.054	0.037	1.450
Quarter 2 Dummy	0.101	0.066	1.520
Quarter 3 Dummy	0.252	0.060	4.200
Gas Price	0.418	0.135	3.080
Constant	3.387	1.385	2.450



Market Segmentation: Aggregate Logit Model

- Consider three alternatives: Air(A), Bus(B), Car(C) for an intercity case

$$D_A = \text{Total demand (number of trips) for A}$$
$$\text{D}_\text{B} = \text{Total demand (number of trips) for B}$$
$$\text{D}_\text{C} = \text{Total demand (number of trips) for C}$$

- Market shares of the alternatives

$$S_A = \text{Ratio of Total demand for } A = D_A/(D_A + D_B + D_C)$$
$$S_B = \text{Total demand for alternative } B = D_B/(D_A + D_B + D_C)$$
$$S_C = \text{Total demand for alternative } C = D_C/(D_A + D_B + D_C)$$

- Consider observed shares as observed (pseudo) probabilities & use Logit function with fixing one alternative as the base alternative (A in this example):
  - Logarithm of the ratio (log of odd-ratio) of the shares become a linear regression model

2-Alternative Aggregate Logit Model

- Consider two alternative urban commuting modes: Car (C_1), Transit (C_2)

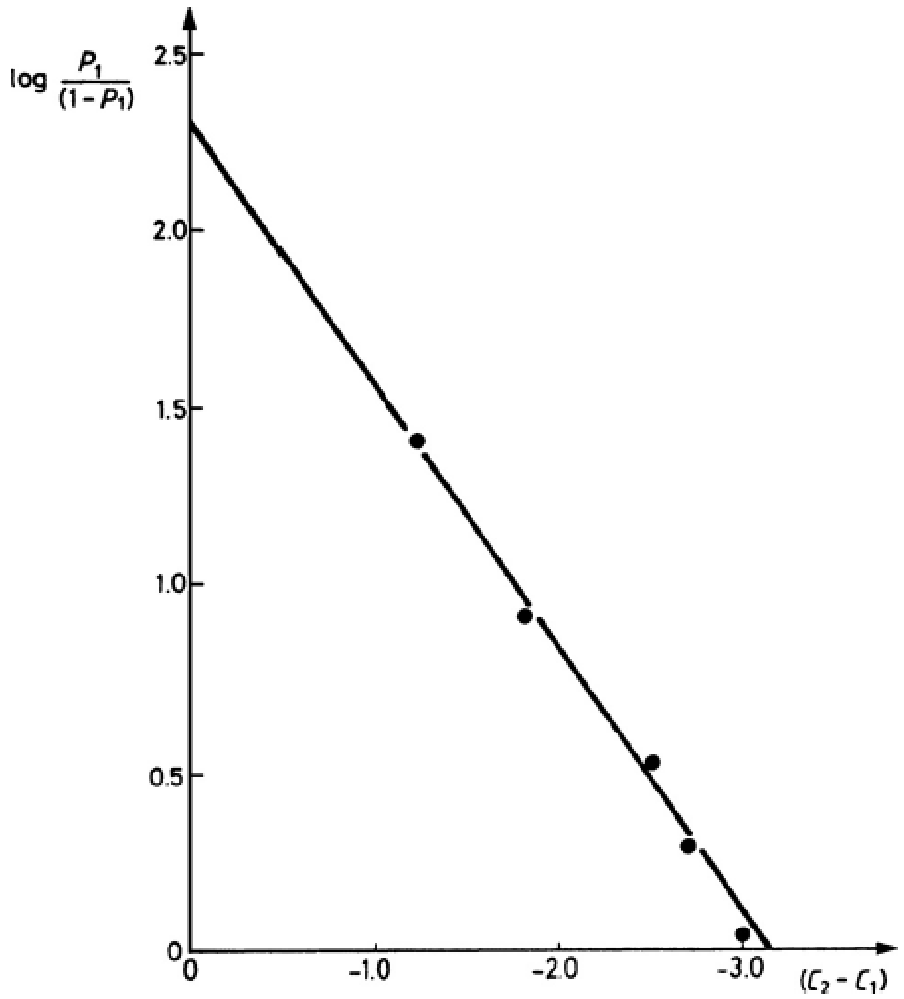
$$Log-Odd\ \ ratio,\ \log\left(\frac{\Pr(transit)}{1-\Pr(transit)}\right)=\beta_0+\beta_c(C_{transit}-C_{car})$$

- Data: Observation

Zone pair	P_1 (%)	P_2 (%)	C_1	C_2	$\log[P_1/(1-P_1)]$
1	51.0	49.0	21.0	18.0	0.04
2	57.0	43.0	15.8	13.1	0.29
3	80.0	20.0	15.9	14.7	1.39
4	71.0	29.0	18.2	16.4	0.90
5	63.0	37.0	11.0	8.5	0.53

2-Alternative Aggregate Logit Model

- Linear-Regression of log-odd ratio against the cost difference will give a model linear logit model



Multi-Alternative Aggregate Logit Model

- Based on indirect utility function and logit formulation (A as the reference alternative)

$$\Pr(C) = S_C = \frac{\exp\left(\sum_c \beta x\right)}{\exp\left(\sum_c \beta x\right) + \exp\left(\sum_B \beta x\right) + 1} \qquad \Pr(B) = S_B = \frac{\exp\left(\sum_B \beta x\right)}{\exp\left(\sum_c \beta x\right) + \exp\left(\sum_B \beta x\right) + 1} \qquad \Pr(A) = S_A = 1 - \Pr(B) - \Pr(C)$$

- Log-odd ratios (with respect to the reference alternative): Can be modelled as Log-linear regression model

$$\log(S_C / S_A) = \sum_C \beta x - 0$$

$$\log(S_A / S_A) = 0$$

$$\log(S_B / S_A) = \sum_B \beta x - 0$$

- Estimation:
  - Maximum likelihood estimation (for more than 2 alternatives)
  - Least square regression as regression of log-odd ratio (convenient for 2 alternatives)

Aggregate Logit Model with Capturing Competition

- Key point is to set the reference alternative. With respect to the reference alternative two types of formulations are possible
  - Ratio of common-attributes (e.g. cost, time) format

$$\ln(S_C / S_A) = \beta_{0-CA} + \sum_{k=1}^{K} \beta_{CA-k} (x_{C-k} / x_{A-k}) + \sum_{j=1}^{J} \gamma_{C-j} x_j$$

$$\ln(S_B / S_A) = \beta_{0-BA} + \sum_{k=1}^{K} \beta_{BA-k} (x_{B-k} / x_{A-k}) + \sum_{j=1}^{J} \gamma_{B-j} x_j$$

  - Difference of common-attributes (e.g. cost, time) format

$$\ln(S_C / S_A) = \beta_{0-CA} + \sum_{k=1}^{K} \beta_{CA-k} (x_{C-k} - x_{A-k}) + \sum_{j=1}^{J} \gamma_{C-j} x_j$$

$$\ln(S_B / S_A) = \beta_{0-BA} + \sum_{k=1}^{K} \beta_{BA-k} (x_{B-k} - x_{A-k}) + \sum_{j=1}^{J} \gamma_{B-j} x_j$$

- Estimation: either by maximum likelihood or least-square for log-odd ratio

- $X_{CA\text{-}k}$ = the common attributes of C and A
- $X_{C\text{-}j}$ = the attributes of only C
- $X_{BA\text{-}k}$ = the common attributes of B and A
- $X_{B\text{-}j}$ = the attributes of only B

Aggregate Logit Model with Capturing Competition

- Elasticity of substitution of a variable between an alternative (B,C) against the reference alternative (A)

$$E_{B-A} = -\beta_{BA}(x_B / x_A) \qquad E_{C-A} = -\beta_{CA}(x_C / x_A)$$

- Elasticity of substitution between two non-reference alternatives (B and C)

$$E_{B-C} = -\beta_{BA}(x_B / x_A) \;\; if \;\; d(x_c) = 0 \;\&\; d(x_B) \neq 0$$
$$E_{B-C} = -\beta_{CA}(x_C / x_A) \;\; if \;\; d(x_c) \neq 0 \;\&\; d(x_B) = 0$$

- Such elasticity measure is problematic as there no consistent measurement of elasticity when attributes of both non-reference alternative change

- Elasticity of substitution depends on reference alternative

Aggregate Logit Model for Capturing Competition

- Elasticity of substitution between two non-reference alternatives (B and C)

$$E_{B-C} = -\beta_{BA}(x_B) \;\; if \;\; d(x_c) = 0 \;\&\; d(x_B) \neq 0$$
$$E_{B-C} = -\beta_{CA}(x_C) \;\; if \;\; d(x_c) \neq 0 \;\&\; d(x_B) = 0$$

- Such elasticity measure is also problematic as there no consistent measurement of elasticity when attributes of both non-reference alternative change

Total Demand & Market Segmentation: Translog Model

- Translog demand system model specifies indirect utility (generalized cost), *V* function of a demand generation process

$$\ln(V_i) = \alpha_0 + \sum_j \alpha_i \ln(\mathrm{Pe}_{ij} / Y_i) + \frac{1}{2}\sum_j \sum_{k \neq j} \beta_{jk} \ln(\mathrm{Pe}_{ij} / Y_i) \ln(\mathrm{Pe}_{ik} / Y_i) + \ldots.$$

- Once specified
  - Total quantity demand for alternative *j*, x_j^* is estimated by applying Roy's Identity,

$$x_{ij}^* = -\left(\partial(V_i) / \partial(Pe_{ij})\right) / \left(\partial(V_i) / \partial(Y_i)\right)$$

  - Finally the demand share of alternative j, $P_i(j)$

$$P_i(j) = \frac{x_{ij}^*}{\sum_j x_{ij}^*}$$

- Pe = Effective price or cost
- i is the individual
- j, k = 1, 2, 3,4, are the alternatives
- Y =Income or Budget
- X_j^* = Total demands of j

Empirical Example: Translog Model

- Modelling demands for making trips of j destinations

$$\ln(V_i) = \alpha_0 + \sum_j \alpha_i \ln(\mathrm{Pe}_{ij} / Y_i) + \frac{1}{2} \sum_j \sum_{k \neq j} \beta_{jk} \ln(\mathrm{Pe}_{ij} / Y_i) \ln(\mathrm{Pe}_{ik} / Y_i) + \ldots.$$

$$x_{ij}^* = \frac{\alpha_j + \sum_k \beta_{jk} \ln(Pe_k / Y)}{(Pe_j / Y)\left(\sum_j \alpha_j + \sum_j \sum_k \beta_{jk} \ln(Pe_k / Y) \right)}$$

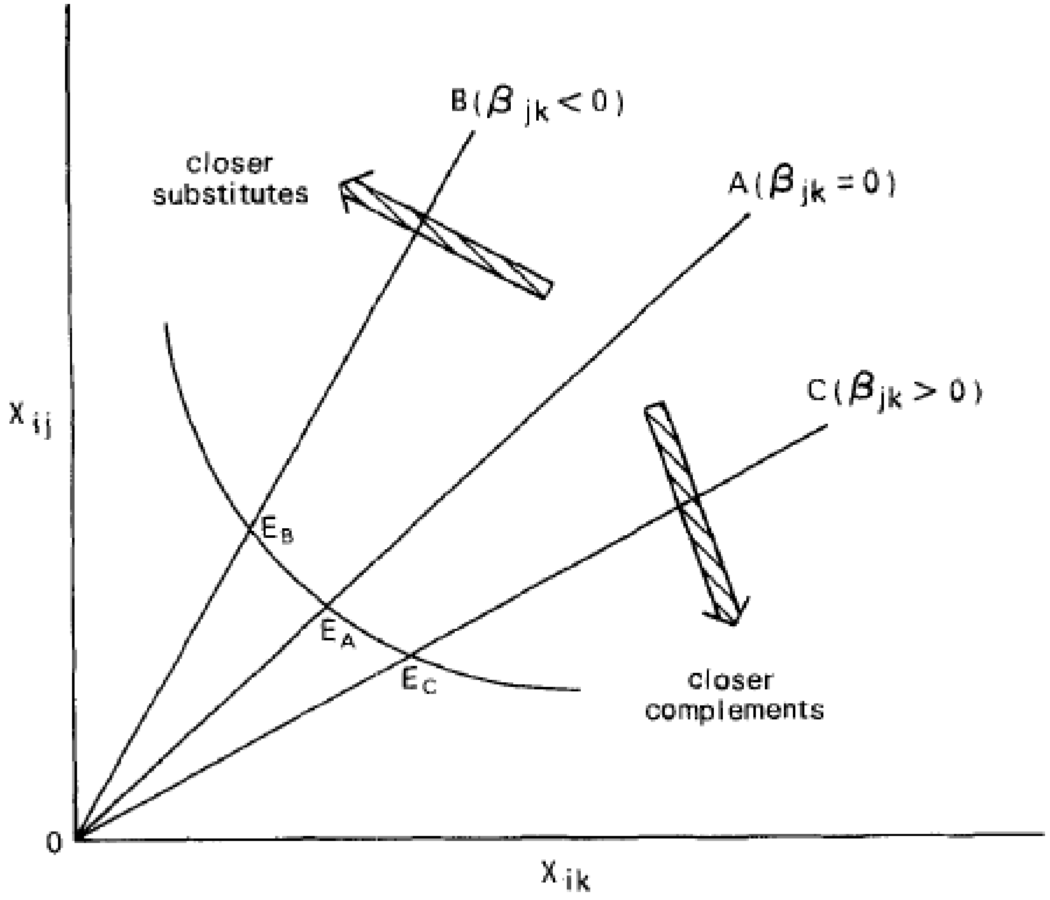
- Pe = Generalized cost of making a trip
- i is the individual
- j, k = 1, 2, 3,4, are the alternatives
- Y =Composite Income / Budget
- X_j^* = Total trip to alternative j

- Assumed that observed demand for alternative j by individual i has, x_{ij}* has randomness and so follows a distribution

x_{ij}^* has lognormally or Possion distribution

- Use Poisson or Lognormal regression model

Translog Model Provides Superior Specification



- Trip generation to alternative destinations (j and k) by an individual (i) at varying substitution scenarios that can be captured by a TransLog model:
  - A zero interaction coefficient (β_{jk}=0) indicates no substitution (independent demands)
  - A negative interaction coefficient (β_{jk}<0) indicates an increase in Trips demand to j at the expense of those to k
  - A positive interaction coefficient (β_{jk}>0) indicates complementary relationship

Market Segmentation: Translog Model

- Translog demand system model is based on microeconomic principle of utility maximization
- Substitution patterns between choice alternatives can be accommodate flexibly
- Model can be estimated by using least-square estimation method or maximum likelihood estimation technique

Predicting Incremental Changes: Elasticity-based Model

- Predicting changes in total demands:

$$T_t = T_{t-1} + E_x \times T_{t-1} \times \frac{x_t - x_{t-1}}{x_{t-1}} \qquad E = \text{Elasticity} = \frac{(T - T_0)/T_0}{(x - x_0)/x_0}$$

  - T_t is the total demand after change of x at time t (e.g. transit ridership at time t)
  - T_{t-1} is the total demand before change of x (e.g. transit ridership time t-1)
  - x_t is the attribute after change at time t (e.g. transit fare at time t)
  - x_{t-1} is the attribute before change at time t (e.g. previous transit fare)

- Elasticity of demand (E) needs to be known and a fixed value
- Useful for short term analysis when the expectation of 'no big change in behaviour' is valid

Predicting Incremental Changes: Elasticity-based Model

- Example of typical functional forms of total demand and corresponding elasticity:

Type	Functional Form	Elasticity
Linear	$T = \alpha + \beta S$	$E = \frac{\beta S}{T} = \frac{1}{1 + \alpha/\beta S}$
Product	$T = \alpha S^{\beta}$	$E = \beta$
Exponential	$T = \alpha \exp(\beta S)$	$E = \beta S$
Share	$p_i = \frac{T_i}{\sum_j T_j}$	$E_{S_i}(p_i) = 1 - p_i$ $E_{S_j}(p_i) = -p_j$

Predicting Incremental Changes: Pivot-Point Model

$$P^{/}(j)=\frac{P^{0}(j)\exp(v_j-v_j^0)}{\sum_k P^{0}(k)\exp(v_k-v_k^0)}$$

- $P^{/}(j)$ is probability/proportion of choosing *j* after change in systematic utility v^0_j
- $P^0(j)$ is probability/proportion of choosing *j* before change
- v_j is the systematic utility function after change
- v^0_j is the systematic utility before change

- Systematic utility function needs to be known and pre-defined
- Considers that preference structure and competition do not change

Aggregate Demand models

- Aggregate models are often very useful:
  - When quick estimation of changes in necessary
  - Lack of detailed micro data for disaggregate modelling
  - Forecasting scenario analysis without precise specification of scenario contexts
- Aggregate and Disaggregate models are complementary:
  - Should not be considered either or

Thank You